Positioning belief functions among uncertainty theories

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The aim of this talk is to show that the theory of belief functions as pioneered by Dempster, introduced by Shafer, and further elaborated by Smets entertains close relationships with other uncertainty theories that now stand as alternatives to Bayesian probability. After insisting on the difference between aleatory and epistemic uncertainty and the difference between frequentist and subjective probability, the presentation will highlight the point that the most elementary representation of epistemic uncertainty takes the form of sets of mutually exclusive values, representing epistemic states. The natural uncertainty theory in that case is possibility theory. Then the talk will propose a general setting for blending possibilistic and probabilistic uncertainty, highlighting major examples of this combination, namely imprecise probabilities, belief functions and numerical possibility theory, in this order of generality. These theories have in common the idea of modelling incomplete information in an unbiased way.

We show that possibility and necessity measures correspond to consonant plausibility and belief functions and that belief functions are lower probabilities with a positive Moebius transform. We exemplify these different notions by comparing p-boxes, fuzzy intervals and interval probability assignments. We place belief functions in the setting of random sets, and comment on the difference between a set-valued random variable and an ill-known random variable. We insist on the difference between the frequentist view of Dempster and the subjectivist view of Shafer on the theory of belief functions. We show that in the continuous cases, possibility distributions, hence consonant belief functions, can be obtained from expert information (subjectivist view), but also from probabilistic inequalities, or yet likelihood functions (statistical view). Finally, we present several partial orders for comparing the informative contents of belief functions, which is another way to connect the three main approaches to epistemic uncertainty.

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