Exercises for the 3rd Belief function school

September 28th - October 2nd, 2015 – Stella plage, France

1 Exercises

1. prove that for any m, $\phi_{pl}(m) \geq \phi_m(m)$

   Solution: Without loss of generality, pick $\omega^* = \arg\max_\omega pl(\omega)$. We then have to proof

$$1 - pl(\omega^*) \geq m(\emptyset),$$

   which can be done by expressing both terms of the inequality as $1 - \sum$ of masses.

2. $$m_1(\{\omega_1, \omega_2\}) = 0.6 \quad m_1(\{\omega_1, \omega_3\}) = 0.4$$
$$m_2(\{\omega_2, \omega_3\}) = 0.5 \quad m_2(\Omega) = 0.5.$$

	$m_1(\{\omega_1, \omega_2\})$	$m_1(\{\omega_1, \omega_3\})$	$\sum_{m_{12}}$
$m_2(\{\omega_2, \omega_3\})$			$= 0.5$
$m_2(\Omega)$			$= 0.5$
$\sum_{m_{12}}$	$= 0.6$	$= 0.4$	

- Are m_1, m_2 weakly/strongly consistent?
- What are the values of κ_m, κ_{pl}?

Solution:

- m_1 and m_2 are weakly consistent (all pairs of focal sets have non-empty intersection), but are not strongly consistent (as the intersection of all focal sets is empty).
- $\kappa_m = [0, 0]$ since m_1, m_2 weakly consistent. $\kappa_{pl} = [0.4, 0.4]$, as the plausibility of each singleton does not depend on the chosen dependence structure, as each ω_i appears either in only one column or only one row of the matrix:

	$m_1(\{\omega_1, \omega_2\})$	$m_1(\{\omega_1, \omega_3\})$	$\sum_{m_{12}}$
$m_2(\{\omega_2, \omega_3\})$	ω_2	ω_3	$= 0.5$
$m_2(\Omega)$	$\{\omega_1, \omega_2\}$	$\{\omega_1, \omega_3\}$	$= 0.5$
$\sum_{m_{12}}$	$= 0.6$	$= 0.4$	

  Finally, the conclusion is that conflict is rather small (although sources disagree on what is the most plausible state), and is not due to a ill-identified dependence.

3. Consider $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ and the masses

$$m_1(\{\omega_1, \omega_2\}) = 0.6 \quad m_1(\{\omega_2, \omega_4\}) = 0.4$$
$$m_2(\{\omega_2, \omega_3\}) = 0.5 \quad m_2(\{\omega_3\}) = 0.5.$$

	$m_1(\{\omega_1, \omega_2\})$	$m_1(\{\omega_2, \omega_4\})$	$\sum_{m_{12}}$
$m_2(\{\omega_2, \omega_3\})$			$= 0.5$
$m_2(\omega_3)$			$= 0.5$
$\sum_{m_{12}}$	$= 0.6$	$= 0.4$	

- Compute the conflicts κ_m, κ_{pl}
- Does independence appear as main cause of conflict?

Solution:

- $\kappa_m = [0.5, 0.5]$ and $\kappa_{pl} = [0.5, 0.5]$ are equal, due again to the fact that ω_2 is in one single row, as well as $\emptyset$:

	$m_1(\{\omega_1, \omega_2\})$	$m_1(\{\omega_2, \omega_4\})$	$\sum_{m_{12}}$
$m_2(\{\omega_2, \omega_3\})$	ω_2	ω_2	$= 0.5$
$m_2(\omega_3)$	$\emptyset$	$\emptyset$	$= 0.5$
$\sum_{m_{12}}$	$= 0.6$	$= 0.4$	

- Both sources disagree, but not because of possible dependence issues. Explanation si that source 1 does not believe ω_3 to be the truth, while source 2 strongly believes so.

4. Consider $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ and the masses

$$m_1(\{\omega_1, \omega_2\}) = 0.6 \quad m_1(\{\omega_1, \omega_4\}) = 0.4$$

$$m_2(\{\omega_2, \omega_3\}) = 0.5 \quad m_2(\omega_3, \omega_4) = 0.5.$$

	$m_1(\{\omega_1, \omega_2\})$	$m_1(\{\omega_1, \omega_4\})$	$\sum_{m_{12}}$
$m_2(\{\omega_2, \omega_3\})$			$= 0.5$
$m_2(\omega_3, \omega_4)$			$= 0.5$
$\sum_{m_{12}}$	$= 0.6$	$= 0.4$	

- Compute the conflicts κ_m, κ_{pl}
- Does independence appear as main cause of conflict?

Solution:

- we have $\kappa_m = [0.1, 0.9]$ and $\kappa_{pl} = [0.5, 0.9]$, with lower bounds obtained by focusing masses outside of $\emptyset$:

	$m_1(\{\omega_1, \omega_2\})$	$m_1(\{\omega_1, \omega_4\})$	$\sum_{m_{12}}$
$m_2(\{\omega_2, \omega_3\})$	$m(\omega_2) = 0.5$	$\emptyset$	$= 0.5$
$m_2(\omega_3, \omega_4)$	$m(\emptyset) = 0.1$	$m(\omega_4) = 0.4$	$= 0.5$
$\sum_{m_{12}}$	$= 0.6$	$= 0.4$	

and the upper bounds by focusing the mass on $\emptyset$.

	$m_1(\{\omega_1, \omega_2\})$	$m_1(\{\omega_1, \omega_4\})$	$\sum_{m_{12}}$
$m_2(\{\omega_2, \omega_3\})$	$m(\omega_2) = 0.1$	$m(\emptyset) = 0.4$	$= 0.5$
$m_2(\omega_3, \omega_4)$	$m(\emptyset) = 0.5$	ω_4	$= 0.5$
$\sum_{m_{12}}$	$= 0.6$	$= 0.4$	

Note, however, that the bounds for κ_m, κ_{pl} are not always obtained for the same assignment of joint masses (as additional exercise, you can try to find such a case).

- In our case, a potentially large part of the conflict may be due to dependence issues between sources, yet not all the conflict can be attributed to this alone (as the lower bounds of κ_m, κ_{pl} are not zero).

5. Given two Bayesian masses m_1, m_2, demonstrate that the lower bounds of κ_m, κ_{pl} are, respectively:

- $1 - \sum_{\omega \in \Omega} \min(pl_1(\omega), pl_2(\omega))$
- $1 - \max_{\omega \in \Omega} \min(pl_1(\omega), pl_2(\omega))$

and can be obtained through the same joint m_{12}.

Solution: sketch) We do have $pl_i(\omega) = m_i(\omega)$, hence maximal plausibilities can only be obtained by focusing most weight on ω such that $\arg\max_{\omega \in \Omega} \min(m_1(\omega), m_2(\omega)$. Since only diagonal elements of the joint square matrix are non-empty, and at most a weight $\min(m_1(\omega), m_2(\omega))$ can be affected to it (due to constraints of the joint mass), then giving $m(\omega_i, \omega_i) = \min(m_1(\omega_i), m_2(\omega_i)$ provides a solution to the problem.

6. Given $\Omega = \{\omega_1, \omega_2, \omega_3\}$, fill in the following table

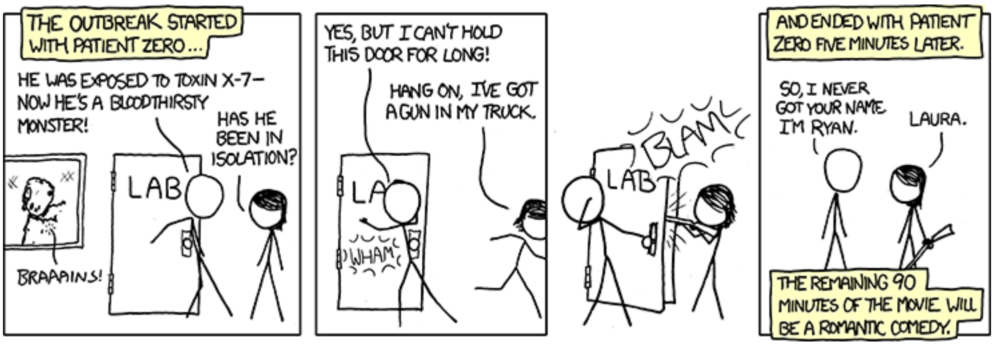
A	m_1	m_2	m_3	$\cup$	2-out-of-3	MCS	$\cap$
$\emptyset$	0	0	0				
$\{\omega_1\}$	0.5	0	0				
$\{\omega_1, \omega_2\}$	0	0.2	0				
$\{\omega_3\}$	0	0	0.6				
$\{\omega_1, \omega_3\}$	0	0	0				
Ω	0.5	0.8	0.4				

Solution: he filled in table is

A	m_1	m_2	m_3	$\cup$	2-out-of-3	MCS	$\cap$
$\emptyset$	0	0	0	0	0	0	0.36
$\{\omega_1\}$	0.5	0	0	0	0.06	0.2	0.2
$\{\omega_1, \omega_2\}$	0	0.2	0	0	0.04	0.04	0.04
$\{\omega_3\}$	0	0	0.6	0	0	0.24	0.24
$\{\omega_1, \omega_3\}$	0	0	0	0	0.24	0.24	0
Ω	0.5	0.8	0.4	1	0.66	0.28	0.16

We can see that those different approaches give different pictures. In particular, the 2-out-of-3 rule does not guarantee $m(\emptyset = 0)$, while MCS and $\cup$ do.

7. A zombie apocalypse has happened, and you must recognize possible threats/supports



The possibilities Ω are

- Zombie (Z)
- Friendly Human (F)
- Hostile Human (H)
- Neutral Human (N)

The sources S_i are

- Half-broken heat detector (S_1)
- Paranoid watch guy 1 (S_2)
- Half-broken Motion detector (S_3)
- Sleepy watch guy 2 (S_4)

	$\hat{\omega}^1 = Z$				$\hat{\omega}^2 = H$				$\hat{\omega}^3 = F$			
	Z	F	H	N	Z	F	H	N	Z	F	H	N
S_1	1	0,5	0,5	0,5	1	0,5	0,5	0,5	0,5	1	1	1
S_2	1	0,2	0,8	0,2	0	0,3	1	0,3	0	0,4	1	0,4
S_3	1	0,5	0,5	0,5	0,5	0,7	0,8	0,7	1	0,5	0,5	0,5
S_4	1	1	1	1	0,2	0,2	1	0,5	0,2	1	0,4	0,8
$\mathbf{w}_1 = (0.5, 0.5, 0, 0)$												
$\mathbf{w}_2 = (0, 0, 0.5, 0.5)$												

Choose $h_{\mathbf{w}_1}$ or $h_{\mathbf{w}_2}$? Given the data, can we find a strictly better weight vector?

Solution: he solution is the following:

	$\hat{\omega}^1 = Z$				$\hat{\omega}^2 = H$				$\hat{\omega}^3 = F$			
	Z	F	H	N	Z	F	H	N	Z	F	H	N
S_1	1	0,5	0,5	0,5	1	0,5	0,5	0,5	0,5	1	1	1
S_2	1	0,2	0,8	0,2	0	0,3	1	0,3	0	0,4	1	0,4
S_3	1	0,5	0,5	0,5	0,5	0,7	0,8	0,7	1	0,5	0,5	0,5
S_4	1	1	1	1	0,2	0,2	1	0,5	0,2	1	0,4	0,8
$\mathbf{w}_1 = (0.5, 0.5, 0, 0)$	**1**	0,35	0,65	0,35	0,5	0,4	**0,75**	0,4	0,25	0,7	**1**	0,7
$\mathbf{w}_2 = (0, 0, 0.5, 0.5)$	**1**	0,75	0,75	0,75	0,35	0,45	**0,9**	0,6	0,6	**0,75**	0,45	0,65

$\mathbf{w}_2$ is preferable to $\mathbf{w}_1$, as it makes no mistakes, while $\sqsupseteq_1$ does make one. On this particular data set, we cannot expect another weight vector $\sqsupseteq$ to perform better, since $\mathbf{w}_2$ make no errors. Note, however, that we may prefer other weight vectors performing as well as $\mathbf{w}_2$ for other reasons (we may prefer non-sparse vectors to sparse ones, that is vectors where as much sources as possible have a positive weight, or the reverse)