BFAS school
Practical session 1

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1 Exercises

1. Represent the uncertainty about the outcome of the Ellsberg's experiment, using a mass function on a suitable frame. Compute the corresponding belief, plausibility and commonality functions.

2. Let $\Omega = \{a; b\}$, and let m and m' be the following mass functions on Ω,
$$m = \{a\}^{\alpha} \oplus \{b\}^{\beta}; \quad m' = \{a\}^{\alpha'} \oplus \{b\}^{\beta'}:$$
   (a) Compute m and m'.
   (b) Compute $m \oplus m'$.
   (c) Compute $m \owedge m'$.

3. Let $[U; V]$ be a random interval. Compute $Bel^{\Theta}((-\infty; u])$ and $Pl^{\Theta}((-\infty; u])$ for any $u \in \mathbb{R}$, using the cdfs of U and V.

4. Let $Y_1; : : : ; Y_n; Z$ be an iid sample from a uniform distribution in $[0; \ ]$. Assume that $\boldsymbol{Y} = (Y_1; : : : ; Y_n)$ is observed and Z is not yet observed.
   (a) Describe the likelihood-based belief function on .
   (b) Describe the predictive belief function on Z.

2 Solutions

1. A suitable frame is $\Omega = \{R; B; Y\}$.

A	$\emptyset$	$\{R\}$	$\{B\}$	$\{Y\}$	$\{R; B\}$	$\{R; Y\}$	$\{B; Y\}$	$\{R; B; Y\}$
$m(A)$	0	1/3	0	0	0	0	2/3	0
$Bel(A)$	0	1/3	0	0	1/3	1/3	2/3	1
$Pl(A)$	0	1/3	2/3	2/3	1	1	2/3	1
$Q(A)$	1	1/3	2/3	2/3	0	0	2/3	0

2. (a) We have

$$m(\{a\}) = \frac{(1- \quad)}{+ \quad -}; \quad m(\{b\}) = \frac{(1- \quad)}{+ \quad -};$$

$$m(\{a; b\}) = \frac{\qquad}{+ \quad -}:$$

A similar expression is obtained for m' by replacing and by $'$ and $'$.

(b) $m \oplus m' = \{a\}^{\alpha\alpha'}\{b\}^{\beta\beta'}$. Consequently,

$$m(\{a\}) = \frac{'(1- \quad ')}{'+ \quad ' - \quad ' \, '}; \quad m(\{b\}) = \frac{'(1- \quad ')}{'+ \quad ' - \quad ' \, '};$$

$$m(\{a; b\}) = \frac{' \quad '}{'+ \quad ' - \quad ' \, '}:$$

(c) $m \textcircled{\wedge} m' = \{a\}^{\min(\alpha,\alpha')}\{b\}^{\min(\beta,\beta')}$. The masses can easily be computed as above.

3. For all $u \in \mathbb{R}$, we have:

$$Bel((-\infty; u]) = \quad ([U; V] \subseteq (-\infty; u]) = \quad (V \leq u) = F_V(u);$$

where F_V is the cumulative distribution function (cdf) of V, and

$$Pl((-\infty; u]) = \quad ([U; V] \cap (-\infty; u] \neq \emptyset) = \quad (U \leq u) = F_U(u):$$

4. (a) The likelihood function is

$$L_{\mathbf{y}}(\ ) = \ ^{-n} \mathbb{1}_{[y_{(n)},+\infty)}(\ );$$

where $y_{(n)} = \max_{1 \leq i \leq n} y_i$, and the contour function is

$$pl_{\mathbf{y}}(\ ) = \left(\frac{y_{(n)}}{\ }\right)^n \mathbb{1}_{[y_{(n)},+\infty)}(\ ):$$

We note that, the contour function being unimodal and upper-semicontinous, the focal sets $\Gamma_{\mathbf{y}}(s)$ are close intervals $[\widehat{\ }_{\mathbf{y}*}(s); \widehat{\ }^{*}_{\mathbf{y}}(s)]$, with $\widehat{\ }_{\mathbf{y}*}(s) = y_{(n)}$ and $\widehat{\ }^{*}_{\mathbf{y}}(s) = y_{(n)} s^{-1/n}$ for all $s \in [0; 1]$. Consequently, the belief function $Bel^{\Theta}_{\mathbf{y}}$ is induced by the random closed interval $[y_{(n)}; y_{(n)} S^{-1/n}]$, with $S \sim \mathcal{U}([0; 1])$.

(b) As $F_\theta(z) = z=$ for all $0 \leq z \leq$, we can write $Z = \ W$ with $W \sim \mathcal{U}([0; 1])$. As function $'(\ ; W) = \ W$ is continuous in , each focal set of $Bel^{\mathbb{Z}}_{\mathbf{y}}$ is an interval

$$'(\Gamma_{\mathbf{y}}(s); w) = [y_{(n)} w; y_{(n)} s^{-1/n} w];$$

so that $Bel^{\mathbb{Z}}_{\mathbf{y}}$ is induced by the random interval

$$[\widehat{Z}_{\mathbf{y}*}; \widehat{Z}^{*}_{\mathbf{y}}] = [y_{(n)} W; y_{(n)} S^{-1/n} W]:$$