BFAS school
Practical session 2

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1st October 2015

1 Exercises

1. (a) In the evidential K-NN rule, the mass function induced by neighbor i is

$$m_i(\{\omega_k\}) = s_{ik}\varphi(d_i), \quad k = 1, \ldots, c$$
$$m_i(\Omega) = 1 - \varphi(d_i),$$

where $s_{ik} = 1$ if neighbor i belongs to class k and $s_{ik} = 0$ otherwise. Compute the contour function $pl(\omega)$ after combining the mass functions from the K nearest neighbors.

   (b) Assume that $\varphi(d_i) = \exp(-\gamma d_i)$. Propose a method for optimizing parameter γ.

2. Let us assume that the complete data $\mathbf{x} = (x_1, \ldots, x_n)$ is a realization from an i.i.d. sample $X_1, \ldots, X_n$ from $\mathcal{B}(\theta)$ with $\theta \in [0, 1]$. We only have partial information about the x_i's in the form: $pl_1, \ldots, pl_n$, where $pl_i(x)$ is the plausibility that $X_i = x$, $x \in \{0, 1\}$.

   (a) Express the cognitive independence assumption.

   (b) Write the expression of the observed-data likelihood.

   (c) Write the E^{2}M algorithm for this problem.

2 Solutions

1. (a) The contour function pl_i corresponding to m_i is

$$pl_j(\omega_k) = \begin{cases} 1 & \text{if } s_{ik} = 1, \\ 1 - \alpha_j & \text{otherwise,} \end{cases}$$
$$= (1 - \alpha_j)^{1 - s_{jk}}$$

for $k = 1, \ldots, c$. The combined contour function is thus

$$pl(\omega_k) \propto \prod_{i \in N_K} (1 - \varphi(d_i))^{1-s_{jk}},$$

for $k = 1, \ldots, c$.

(b) Let pl_{-i} be the contour function about the class of object i, determined from the other objects on the dataset. We should have $pl_{-i}(\omega_k)$ close to 1 if $s_{ik} = 1$ and $pl_{-i}(\omega_k)$ close to 0 otherwise. We can find γ minimizing the following cost function

$$E(\gamma) = \sum_i \sum_k (pl_{-i}(\omega_k) - s_{ik})^2.$$

2. (a) The cognitive assumption is here

$$pl(x_1, \ldots, x_n) = \prod_{i=1}^{n} pl(x_i)$$

for all $(x_1, \ldots, x_n) \in \{0, 1\}^n$.

(b) Under the cognitive independence assumption,

$$\log L(\theta; pl_1, \ldots, pl_n) = \sum_{i=1}^{n} \log \left[(1-\theta) pl_i(0) + \theta pl_i(1) \right].$$

(c) The complete data log-likelihood is

$$\log L(\theta, \mathbf{x}) = n \log(1-\theta) + \log\left(\frac{\theta}{1-\theta}\right) \sum_{i=1}^{n} x_i.$$

E-step: compute

$$Q(\theta, \theta^{(q)}) = n \log(1-\theta) + \log\left(\frac{\theta}{1-\theta}\right) \sum_{i=1}^{n} \xi_i^{(q)}, \text{ with}$$

$$\xi_i^{(q)} = \mathbb{E}_{\theta^{(q)}}\left[X_i | pl_i\right] = \frac{\theta^{(q)} pl_i(1)}{(1-\theta^{(q)}) pl_i(0) + \theta^{(q)} pl_i(1)}.$$

M-step:

$$\theta^{(q+1)} = \frac{1}{n} \sum_{i=1}^{n} \xi_i^{(q)}.$$